Eur. Phys. J. D **37**, 451–456 (2006)
DOI: 10.1140/epjd/e2005-00315-2

THE EUROPEAN PHYSICAL JOURNAL D

CNOT gate by adiabatic passage with an optical cavity

N. Sangouard[1,a], X. Lacour[2], S. Guérin[2], and H.R. Jauslin[2]

[1] Fachbereich Physik, Universität Kaiserslautern, Erwin-Schrödinger-Strasse, 67663 Kaiserlsautern, Germany
[2] Laboratoire de Physique, Université de Bourgogne, UMR CNRS 5027, B.P. 47870, 21078 Dijon Cedex, France

Received 24 May 2005 / Received in final form 26 September 2005
Published online 22 November 2005 –

Abstract. We propose a scheme for the construction of a CNOT gate by adiabatic passage in an optical cavity. In opposition to a previously proposed method, the technique is not based on fractional adiabatic passage, which requires the control of the ratio of two pulse amplitudes. Moreover, the technique constitutes a decoherence-free method in the sense that spontaneous emission and cavity damping are avoided since the dynamics follows dark states.

PACS. 03.67.Lx Quantum computation – 32.80.Qk Coherent control of atomic interactions with photons

1 Introduction

The controlled-not (CNOT) gate acts on systems composed of two qubits. The first qubit controls the not operation on the second (target) qubit: if the control qubit is in state $|0\rangle$, the target keeps its state whereas if the control is in state $|1\rangle$, the state of the target is switched. The set composed of the CNOT gate and of elementary one-qubit gates forms a universal set, i.e. all logical gates can be constructed by the composition of gates in this set [1]. The CNOT gate allows to prepare entangled states from factorizable superposition states. Entanglement is a key ingredient of quantum computation [2], quantum teleportation [3] or secure quantum cryptography [4] and thus confers to the CNOT gate a broad practical interest.

The efficient treatment of quantum information requires qubits insensitive to decoherence, easily prepared and measured. Furthermore, the gates operating on the qubits have to be robust with respect to variations or partial knowledge of experimental parameters. These requirements can be satisfied if the quantum information is represented by atomic states controlled by adiabatic fields. Indeed, the decoherence due to spontaneous emission can be avoided if the dynamics follows dark states, i.e. states without components on lossy excited states. In this context, a mechanism has been proposed in reference [5] to implement by adiabatic passage all one-qubit gates, i.e. a general unitary matrix U in SU(2). A tripod-type system [6] is used and as in fractional stimulated Raman adiabatic passage (f-STIRAP) [7], the amplitudes of two pulses are required to have a constant ratio. The realisation of this technique requires a specific system (for instance a system of Zeeman states) to be robust [7]. In a scheme, first introduced in reference [8], composed of atoms fixed inside a single-mode optical cavity, a mechanism has been proposed [9] for the creation of a two-qubit controlled-phase (C-*phase*) gate by stimulated Raman adiabatic passage (STIRAP) processes [10,11] and a two-qubit controlled-unitary (C-U) gate requiring f-STIRAP processes. Additionally, it was suggested in reference [9] to work with five-level systems composed of three ground states (two of them are the qubit states, the other one an ancillary state) and of two excited states never populated in the adiabatic limit. The proposal was to use the two excited states to realise the Raman transitions involved respectively in the construction of one-qubit gates and the C-*phase* gate. This gives a technique for the preparation of the universal set $\{U$, C-*phase*$\}$ [12] and all logical quantum gates can thus be obtained from the composition of these two gates. The construction of a CNOT gate from the universal set $\{U$, C-*phase*$\}$ or from the C-U gate requires the control of the ratio between two pulse amplitudes since f-STIRAP is used in both methods. A five-level system in which the transitions can be excited independently and the ratio of the pulsed fields can be controlled robustly has to be found.

In this paper, we adapt to the preparation of the CNOT gate an alternative mechanism based on adiabatic passage along dark states that was used to construct directly the SWAP gate [13]. The mechanism is only based on STIRAP processes. It can therefore be implemented robustly in a variety of systems, avoiding e.g. the requirement encountered in other schemes of using very specific Zeeman-sublevels. Moreover, it constitutes a decoherence-free method in the sense that in the adiabatic limit, the excited atomic states and the cavity mode (in the limit of a cavity Rabi frequency much larger than the laser Rabi frequency) are negligibly populated during the dynamics.

[a] e-mail: nicolas.sangouard@physik.uni-kl.de

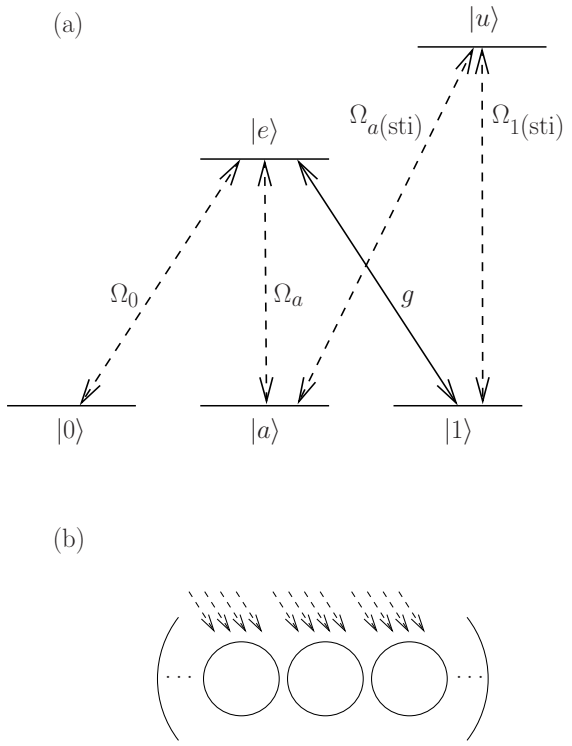

Fig. 1. (a) Schematic representation of the five-level atom. The laser (cavity) couplings are represented by dashed (full) arrows. (b) Representation of the atomic register trapped in a single-mode optical cavity. The atoms are represented by circles, laser fields by arrows.

Furthermore, we also show that the proposed mechanism can be used to directly prepare some specific composed gates. The usual technique to construct a specific gate consists generally in combining elementary gates belonging to a universal set. Since in the experimental realisation of each gate there always are uncontrollable losses, it is usefull to design instead direct implementations of specific compositions of elementary gates.

We present the atomic configuration associated to the qubits in Section 2. In Section 3, we develop the mechanism and the analytical calculations of the instantaneous eigenstates adiabatically involved in the dynamics. In Section 4, we show the result of numerical simulations. Before concluding, we extend the mechanism allowing to build the CNOT gate to the direct generation of specific composed gates.

2 The system

Although the mechanism could be realised in a system composed of non-degenerate ground states, we use the five-level atomic system as presented in Figure 1a, in which other gates have also been implemented [9]. The three ground states $|0\rangle$, $|a\rangle$ and $|1\rangle$ are coupled to the excited state $|e\rangle$ respectively by two lasers (associated to the Rabi frequencies Ω_0 and Ω_a), and by a single mode cavity (associated to the Rabi frequency g). Furthermore, $|a\rangle$ and $|1\rangle$ are coupled by two additional lasers (with Rabi frequencies $\Omega_{a(\mathrm{sti})}$ and $\Omega_{1(\mathrm{sti})}$) to the upper state $|u\rangle$. The polarizations and the frequencies are such that each field drives a unique transition. The atomic states $|0\rangle$ and $|1\rangle$ represent the computational states of the qubit. We consider that the atomic register is fixed in the single-mode optical cavity as represented in Figure 1b. Each atom (labeled by k) of the register is driven by a set of four pulsed laser fields $\Omega_0^{(k)}(t)$, $\Omega_a^{(k)}(t)$, $\Omega_{a(\mathrm{sti})}^{(k)}$ and $\Omega_{1(\mathrm{sti})}^{(k)}$ and by the cavity mode $g^{(k)}$ which is time independent.

3 The mechanism

3.1 General strategy

We first recall how the CNOT gate acts. Before the interaction with the lasers, the initial state $|\psi_i\rangle$ of the atoms in the cavity is defined as

$$|\psi_i\rangle = \alpha|00\rangle|0\rangle + \beta|01\rangle|0\rangle + \gamma|10\rangle|0\rangle + \delta|11\rangle|0\rangle, \quad (1)$$

where the labels s_1, s_2 of the states of the form $|s_1 s_2\rangle|0\rangle$ denote respectively the states of the first and second atom, and $|0\rangle$ is the initial vacuum state of the cavity-mode field. $\alpha, \beta, \gamma, \delta$ are complex coefficients. The CNOT gate exchanges the states $|0\rangle$ and $|1\rangle$ of the second target qubit when the first control qubit is in state $|1\rangle$ leading to the output state

$$|\psi_o\rangle = \alpha|00\rangle|0\rangle + \beta|01\rangle|0\rangle + \gamma|11\rangle|0\rangle + \delta|10\rangle|0\rangle. \quad (2)$$

We use a simple interaction scheme to represent the proposed mechanism for the creation of a CNOT gate (see Fig. 2). This mechanism is composed of six steps. Since the state $|11\rangle|0\rangle$ is a stationary state (if there are no photons in the cavity there cannot be any transition from $|1\rangle$ to $|e\rangle$), we first transfer the population of the state $|1\rangle$ of the second atom into the ancillary state $|a\rangle$ by STIRAP. The next four steps allow to swap the populations of the states $|10\rangle|0\rangle$ and $|1a\rangle|0\rangle$. The last step transfers back the population of the ancillary state $|a\rangle$ of the second atom into the state $|1\rangle$. The population transfers are realised by adiabatic passage along dark states (i.e. with no components in the atomic excited states and a negligible component in the excited cavity states). We thus obtain a decoherence-free method for the creation of the CNOT gate. In the next subsection, we give details of each step.

3.2 Description of the steps

The six steps summarized above are obtained as follows.

Step 1. The population of state $|1\rangle$ of the second atom is completely transferred into $|a\rangle$ by the use of two resonant pulses $\Omega_{a(\mathrm{sti})}^{(2)}$, $\Omega_{1(\mathrm{sti})}^{(2)}$, with relative phase $\varphi = \pi$, switched on and off in a counterintuitive pulse sequence (i.e. $\Omega_{a(\mathrm{sti})}^{(2)}$ before $\Omega_{1(\mathrm{sti})}^{(2)}$.) After this STIRAP interaction [10,11], the initial state (1) becomes

$$|\psi_1\rangle = \alpha|00\rangle|0\rangle + \beta|0a\rangle|0\rangle + \gamma|10\rangle|0\rangle + \delta|1a\rangle|0\rangle. \quad (3)$$

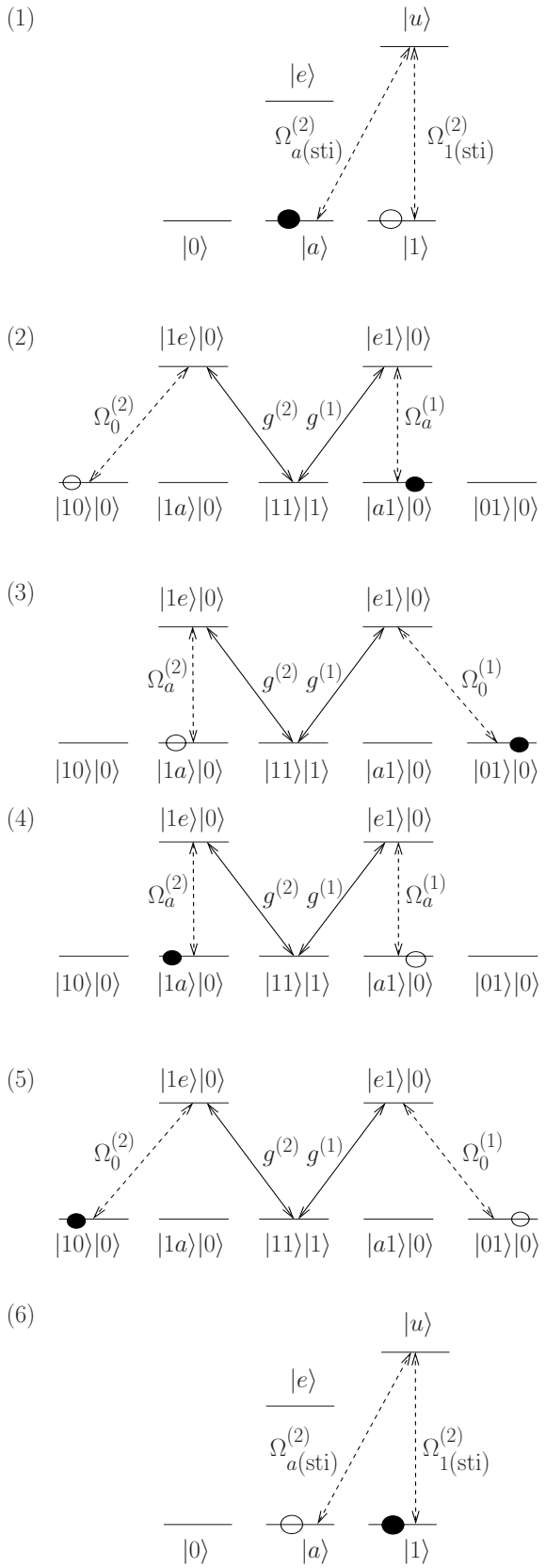

Fig. 2. Schematic representation of the six steps of the construction of the CNOT gate. For each step, the initial state is represented by an empty circle whereas the final state is symbolized by a full black circle.

Step 2. The population of state $|10\rangle|0\rangle$ is transferred into $|a1\rangle|0\rangle$ with the use of the sequence $\Omega_a^{(1)}$, $\Omega_0^{(2)}$. This adiabatic transfer is a non-trivial coherent process described in [8]. It uses a five-level extended STIRAP with constant intermediate couplings. The state (3) reads

$$|\psi_2\rangle = \alpha|00\rangle|0\rangle + \beta|0a\rangle|0\rangle + \gamma|a1\rangle|0\rangle + \delta|1a\rangle|0\rangle. \quad (4)$$

Step 3. With a similar technique, the population of $|1a\rangle|0\rangle$ is transferred into $|01\rangle|0\rangle$ by the use of the counterintuitive sequence of the two pulses $\Omega_0^{(1)}$, $\Omega_a^{(2)}$ leading to the state

$$|\psi_3\rangle = \alpha|00\rangle|0\rangle + \beta|0a\rangle|0\rangle + \gamma|a1\rangle|0\rangle + \delta|01\rangle|0\rangle. \quad (5)$$

Step 4. With a similar technique, the population of $|a1\rangle|0\rangle$ is transferred into $|1a\rangle|0\rangle$ by the use of the sequence $\Omega_a^{(2)}$, $\Omega_a^{(1)}$ giving

$$|\psi_4\rangle = \alpha|00\rangle|0\rangle + \beta|0a\rangle|0\rangle + \gamma|1a\rangle|0\rangle + \delta|01\rangle|0\rangle. \quad (6)$$

Step 5. With a similar technique, the population of $|01\rangle|0\rangle$ is transferred into $|10\rangle|0\rangle$ by the use of the sequence $\Omega_0^{(2)}$, $\Omega_0^{(1)}$ in such a way that the state (6) becomes

$$|\psi_5\rangle = \alpha|00\rangle|0\rangle + \beta|0a\rangle|0\rangle + \gamma|1a\rangle|0\rangle + \delta|10\rangle|0\rangle. \quad (7)$$

Step 6. The population of the state $|a\rangle$ of the second atom is transferred back by STIRAP into $|1\rangle$ by the use of the sequence of pulses $\Omega_{1(\mathrm{sti})}^{(2)}$, $\Omega_{a(\mathrm{sti})}^{(2)}$ with relative phase $\varphi = \pi$. As a result, the system is in state

$$|\psi_6\rangle = \alpha|00\rangle|0\rangle + \beta|01\rangle|0\rangle + \gamma|11\rangle|0\rangle + \delta|10\rangle|0\rangle, \quad (8)$$

which coincides with the output state of the CNOT gate.

3.3 Calculation of the instantaneous eigenstates

We calculate the instantaneous eigenvectors connected with the initial condition and that are thus adiabatically followed by the dynamics when the two atoms interact with two laser fields and the cavity-mode. We show that they are dark states with no component in the atomic excited states and a negligible component in the excited cavity states.

We give the details of steps (1)-(6) first and next (2)-(3)-(4)-(5).

The steps (1) and (6) are the well known STIRAP process [10,11]. The dynamics follows the dark state

$$|\phi_{(\mathrm{sti})}\rangle \propto \Omega^{(2)}_{\mathrm{a(sti)}}|1\rangle - e^{i\varphi}\Omega^{(2)}_{1(\mathrm{sti})}|a\rangle \quad (9)$$

(where φ is the relative phase of the pulses $\Omega^{(2)}_{1(\mathrm{sti})}$ and $\Omega^{(2)}_{\mathrm{a(sti)}}$) that transfers population from $|1\rangle$ ($|a\rangle$) to $|a\rangle$ ($|1\rangle$) with a counterintuitive pulse sequence. We choose the phase $\varphi = \pi$ to avoid a minus sign on the states $|a\rangle$ and $|1\rangle$ of the second atom after the steps (1) and (6) respectively.

Concerning the intermediate steps, since the lasers do not couple the atomic state $|1\rangle$, the state $|\phi_1^{(1)}\rangle = |11\rangle|0\rangle$ (defining one dimensional Hilbert space $\mathcal{H}_1$) of the initial condition (1) is decoupled from the other ones. The other states of (1) are connected to two orthogonal decoupled subspaces denoted $\mathcal{H}_7$ and $\mathcal{H}_{16}$, respectively spanned by the states

$$\mathcal{H}_7 = \{|01\rangle|0\rangle, |10\rangle|0\rangle, |1a\rangle|0\rangle, |a0\rangle|0\rangle, |1e\rangle|0\rangle, |e1\rangle|0\rangle, |11\rangle|1\rangle\}, \quad (10)$$

and

$$\mathcal{H}_{16} = \{|00\rangle|0\rangle, |0a\rangle|0\rangle, |01\rangle|1\rangle, |0e\rangle|0\rangle, |a0\rangle|0\rangle, |aa\rangle|0\rangle, |a1\rangle|1\rangle, |ae\rangle|0\rangle, |1e\rangle|1\rangle, |e0\rangle|0\rangle, |10\rangle|1\rangle, |1a\rangle|1\rangle, |11\rangle|2\rangle, |ea\rangle|0\rangle, |e1\rangle|1\rangle, |ee\rangle|0\rangle\}. \quad (11)$$

For each step, one ground state $|0\rangle$ or $|a\rangle$ of each atom is coupled by a laser field to the excited state, while the other one is not coupled to the excited state. To describe the calculation of the instantaneous eigenstates for the four steps, we introduce the following notation: the state coupled by a laser field is labeled $|L^{(i)}\rangle$ ($|0^{(i)}\rangle$ or $|a^{(i)}\rangle$) and the non-coupled state $|N^{(i)}\rangle$ ($|a^{(i)}\rangle$ or $|0^{(i)}\rangle$). The index $i = 1, 2$ labels the atom i. In the full Hilbert space $\mathcal{H} = \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{F}$ with $\mathcal{H}^{(i)}$ the Hilbert space associated to the atom i and $\mathcal{F}$ the Fock space, the Hamiltonian (in units such that $\hbar = 1$) reads in the rotating wave approximation

$$\begin{aligned} H(t) = {} & \omega_c a^\dagger a + \omega_e |e^{(1)}\rangle\langle e^{(1)}| + \omega_e |e^{(2)}\rangle\langle e^{(2)}| \\ & + \left(\Omega^{(1)}(t) e^{-i\omega t} |e^{(1)}\rangle\langle L^{(1)}| + g^{(1)} a |e^{(1)}\rangle\langle 1^{(1)}| + \text{h.c.}\right) \\ & + \left(\Omega^{(2)}(t) e^{-i\omega t} |e^{(2)}\rangle\langle L^{(2)}| + g^{(2)} a |e^{(2)}\rangle\langle 1^{(2)}| + \text{h.c.}\right) \end{aligned} \quad (12)$$

where a ($a^\dagger$) is the annihilation (creation) operator for the cavity mode, ω (ω_c) is the frequency of the laser field (cavity mode) and ω_e is the energy of the excited state (the energy reference is taken for the ground states: $\omega_0 = \omega_a = \omega_1 = 0$). We consider resonant fields: $\omega_e = \omega = \omega_c$. $\Omega^{(i)}(t)$ and $g^{(i)}$ are the Rabi frequencies associated to the laser pulse and to the cavity respectively for the atom i. ($\Omega^{(i)}(t)$ corresponds to $\Omega_0^{(i)}(t)$ or $\Omega_a^{(i)}(t)$, depending on the ground state $|0\rangle$ or $|a\rangle$ of the atom i coupled by the laser.) The dynamics is determined by the Schrödinger equation $i\partial\psi(t)/\partial t = H(t)\psi(t)$. The Hamiltonian in the interaction picture

$$H_{\text{I}}(t) = T^\dagger(t) H(t) T(t) - i T^\dagger(t) \frac{dT}{dt}(t) \quad (13a)$$

with

$$T(t) = e^{-i\omega t\left(a^\dagger a + |e^{(1)}\rangle\langle e^{(1)}| + |e^{(2)}\rangle\langle e^{(2)}|\right)} \quad (13b)$$

reads

$$\begin{aligned} H_{\text{I}}(t) = {} & \Omega^{(1)}(t) |e^{(1)}\rangle\langle L^{(1)}| + g^{(1)} a |e^{(1)}\rangle\langle 1^{(1)}| \\ & + \Omega^{(2)}(t) |e^{(2)}\rangle\langle L^{(2)}| + g^{(2)} a |e^{(2)}\rangle\langle 1^{(2)}| \\ & + \text{h.c.} \end{aligned} \quad (14)$$

The dynamics is therefore determined by

$$i\frac{\partial}{\partial t}\phi(t) = H_{\text{I}}(t)\phi(t) \quad (15)$$

with

$$\psi(t) = T(t)\phi(t). \quad (16)$$

We remark that the transformation T does not change the initial state (1). The Hamiltonian $H_{\text{I}}(t)$ is block-diagonal; three blocks connected to the initial state (1) have to be considered:

$$H_{\text{I}}(t) = \begin{bmatrix} H_1 = 0 & 0 & 0 \\ 0 & H_7(t) & 0 \\ 0 & 0 & H_{16}(t) \end{bmatrix} \quad (17)$$

with $H_d(t)$ acting in the d-dimensional subspace $\mathcal{H}_d$ generated by the set of states defined in equations (10) and (11). The adiabatic evolution of the initial state (1) is completely described by the dark states of the Hamiltonian H_{I}, labeled $\phi_d^{(k)}(t) \in \mathcal{H}_d$ (with k the index of degeneracy of $\mathcal{H}_d$). These dark states are instantaneous eigenstates that don't have any components on the atomic excited states. They are associated to null eigenvalues. Although these dark states are degenerate, they evolve without any geometric phase. One can easily check that all the elements contributing to this geometric phase [14], $\langle\phi_d^{(k')}(s)|d/ds|\phi_d^{(k)}(s)\rangle$, are null during the dynamics since for $k = k'$, the phase of the lasers is constant for each step (as in standard STIRAP) and for $k \neq k'$, the dark states belong to orthogonal subspaces. Therefore, according to the adiabatic theorem, the dynamics follows the dark states initially connected to each component of the initial state (1)

$$|\psi(t)\rangle \approx T(t) \sum_{d,k} c_d^k |\phi_d^{(k)}(t)\rangle, \quad (18)$$

with the coefficients

$$\begin{aligned} c_d^k &= \left\langle \phi_d^{(k)}(t_i) | T^\dagger(t_i) | \psi(t_i) \right\rangle \\ &= \left\langle \phi_d^{(k)}(t_i) | \psi(t_i) \right\rangle. \end{aligned} \quad (19)$$

We have thus to determine the instantaneous eigenstates. In the subspace $\mathcal{H}_7$, the states $|\phi_7^{(1)}\rangle = |N^{(1)}1\rangle|0\rangle$ and $|\phi_7^{(2)}\rangle = |1N^{(2)}\rangle|0\rangle$ are not coupled to the initial state (1) and do not participate in the dynamics. Only the atomic dark state [8]:

$$\begin{aligned} |\phi_7^{(3)}\rangle \propto {} & g^{(1)}\Omega^{(2)} |L^{(1)}1\rangle|0\rangle + g^{(2)}\Omega^{(1)} |1L^{(2)}\rangle|0\rangle \\ & - \Omega^{(1)}\Omega^{(2)} |11\rangle|1\rangle \end{aligned} \quad (20)$$

(where the normalisation coefficient has been omitted) participates to the dynamics. The second step, associated to $L^{(1)} \equiv a$, $L^{(2)} \equiv 0$, $\Omega^{(1)} \equiv \Omega_a^{(1)}$, $\Omega^{(2)} \equiv \Omega_0^{(2)}$ leads to the initial and final connections symbolically written as $|10\rangle|0\rangle \rightarrow |\phi_7^{(3)}\rangle \rightarrow |a1\rangle|0\rangle$ (see Fig. 3). The third,

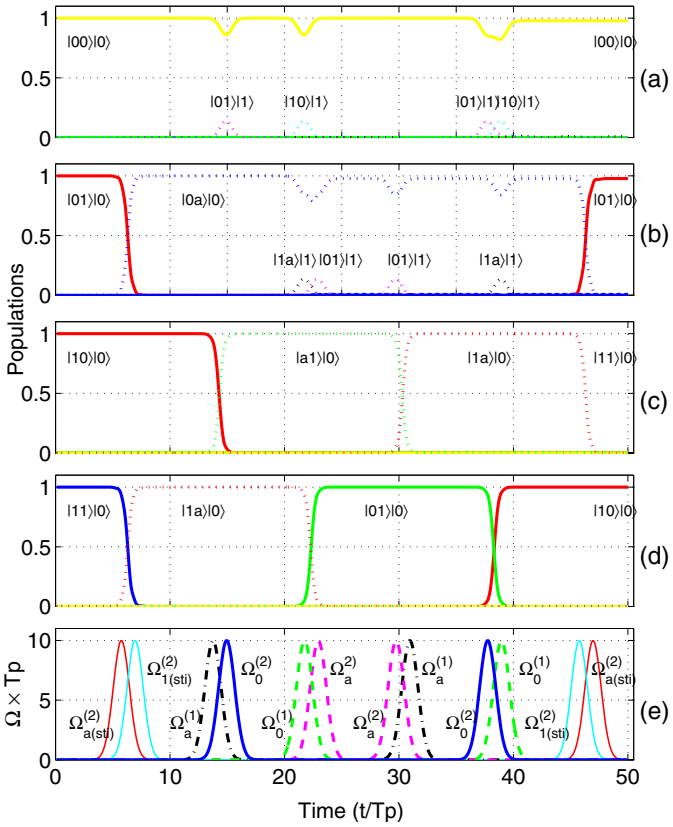

Fig. 3. Temporal evolution of initial conditions (a) $|00\rangle|0\rangle$, (b) $|01\rangle|0\rangle$, (c) $|10\rangle|0\rangle$ and (d) $|11\rangle|0\rangle$. The states which are populated during the interaction with the pulses or between two steps are indicated. (e) Temporal profile of the Rabi frequencies. The parameters used are $\Omega_{\max}T_p = 10$, $gT_p = 25$. The delay between two pulses in a step is $1.2T_p$. A colour version of the figure is available in electronic form at `http://www.eurphysj.org`.

fourth and fifth steps give respectively the connections $|1a\rangle|0\rangle \rightarrow |\phi_7^{(3)}\rangle \rightarrow |01\rangle|0\rangle$, $|a1\rangle|0\rangle \rightarrow |\phi_7^{(3)}\rangle \rightarrow |1a\rangle|0\rangle$, and $|01\rangle|0\rangle \rightarrow |\phi_7^{(3)}\rangle \rightarrow |10\rangle|0\rangle$. We determine four atomic dark states in the subspace $\mathcal{H}_{16}$ connected to the component $|00\rangle|0\rangle$ of the initial condition (1):

$$|\phi_{16}^{(2)}\rangle \propto \Omega^{(2)}|N^{(1)}1\rangle|1\rangle - g^{(2)}|N^{(1)}L^{(2)}\rangle|0\rangle \quad (21a)$$

$$|\phi_{16}^{(3)}\rangle \propto \Omega^{(1)}|1N^{(2)}\rangle|1\rangle - g^{(1)}|L^{(1)}N^{(2)}\rangle|0\rangle, \quad (21b)$$

$$|\phi_{16}^{(4)}\rangle = |N^{(1)}N^{(2)}\rangle|0\rangle, \quad (21c)$$

$$|\phi_{16}^{(5)}\rangle \propto g^{(1)}g^{(2)}\sqrt{2}|L^{(1)}L^{(2)}\rangle|0\rangle - g^{(2)}\Omega^{(1)}\sqrt{2}|1L^{(2)}\rangle|1\rangle - g^{(1)}\Omega^{(2)}\sqrt{2}|L^{(1)}1\rangle|1\rangle + \Omega^{(1)}\Omega^{(2)}|11\rangle|2\rangle. \quad (21d)$$

We remark that the state $|00\rangle|0\rangle$ is connected initially and finally to the dark state $|\phi_{16}^{(n)}\rangle$ at the nth step. Since the dynamics follows atomic dark states, the excited atomic state is never populated (in the adiabatic limit). Moreover, the projections of the dark states on the excited cavity photon states can be made negligible if $g^{(i)} \gg \Omega^{(i)}$ [15]. In this case, the mechanism we propose is a decoherence-free method in the sense that the process is not sensitive to spontaneous emission from the atomic excited states nor to the lifetime of photons in the optical cavity.

4 Numerical validation

We present the numerical validation of the mechanism proposed for the construction of the CNOT gate.

We show in Figure 3 the time evolution of four initial states: in (a) and (b) the population of initial states $|00\rangle|0\rangle$ and $|01\rangle|0\rangle$ respectively stays in these states after the interaction with the twelve pulses since the control qubit is in state $|0\rangle$, in (c) and (d) the population of initial states $|01\rangle|0\rangle$ and $|11\rangle|0\rangle$ are exchanged. In (e), we show the Rabi frequencies associated to the pulses. The laser Rabi frequencies are all chosen of the form $\Omega(t) = \Omega_{\max}e^{-(t/T_p)^2}$. The steps (1) and (6) of the mechanism can be explained by the standard STIRAP technique [10,11]. The other steps involve the two atoms and the cavity using an adiabatic transfer which is a five-level extended STIRAP with constant intermediate couplings [8]. The couplings have to satisfy $\Omega_{\max}T_p, gT_p \gg 1$ to fulfill the adiabatic conditions. The delay between two pulses of the same step is chosen equal to $1.2\,T_p$ to minimize the non-adiabatic losses [7]. Moreover, the condition $g \gg \Omega_{\max}$ has to be satisfied such that the cavity mode is negligibly populated during the interaction with the pulses.

5 Discussion

In the optical domain, one can give an estimate of the relevant parameters. Taking into account the losses of the cavity (characterized by the decay rate κ of the cavity field) and of the excited states (of lifetime τ), we have to satisfy the adiabatic conditions: $\Omega_{\max}T_p, gT_p \gg 1$ and $(\Omega_{\max}T_p)^2, (gT_p)^2 \gg \kappa T_p, T_p/\tau$. The latter is satisfied for $g, \Omega_{\max} \gg \kappa, 1/\tau$. For a typical pulse duration of $T_p = 50$ ns, we use $\Omega_{\max} = 1.2 \times 10^8$ s^{-1}, which is achievable experimentally (see for instance Ref. [16]). We use a cavity coupling $g = 5.2 \times 10^8$ s^{-1}, more than four times larger than $\Omega_{\max}$ to have a small population in the cavity field. Such a strong coupling has been recently achieved in experiments with atoms in an optical cavity trapped with a duration of the order of one second [17,18]. For a realistic decay rate ($\kappa = 1 \times 10^7$ s^{-1}) of the cavity, the numerical simulation of the proposed process gives: (i) 80% of the population of states $|00\rangle|0\rangle$ and $|01\rangle|0\rangle$ are left on these states and (ii) the exchange of the population between $|10\rangle|0\rangle$ and $|11\rangle|0\rangle$ is of the order of 90%. For a decay rate ($\kappa = 1 \times 10^6$ s^{-1}), we would obtain that 96% of the population of the states $|00\rangle|0\rangle$ and $|01\rangle|0\rangle$ are preserved, and 92% of the population of $|10\rangle|0\rangle$ and $|11\rangle|0\rangle$ are exchanged. This analysis shows that the mechanism could be implemented with an observable efficiency with the currently available technology. Longer cavity photon lifetimes and/or larger cavity couplings would give a very good efficiency.

We remark that the pulse $\Omega_a^{(2)}$ is used two times successively in the steps (3) and (4). These two pulses can thus be replaced by a single pulse. The process then requires the use of only eleven pulses.

By manipulating the phase of the pulses, the technique proposed in this paper can be extend to the direct preparation of the composition of elementary gates. Indeed, if instead of taking a phase difference equal to π between the first and the second laser for the steps (1) and (6) and zero otherwise, we add an arbitrary relative phase $\varphi_{(n)}$ in step (n), the proposed mechanism leads to the following gate composition:

$$Ph^{(2)}(\varphi_{(6)}) \circ \text{C-}phase(\varphi_{(2)} + \varphi_{(4)}) \circ \text{CNOT} \circ \text{C-}phase(\varphi_{(3)} + \varphi_{(5)}) \circ Ph^{(2)}(\varphi_{(1)}). \quad (22)$$

Similarly, the technique we proposed to build the SWAP gate in reference [13] leads to the composition

$$Ph^{(1)}(\alpha) \circ \text{SWAP} \circ Ph^{(1)}(\beta) \circ \text{C-}phase(-\alpha - \beta) \quad (23)$$

where α, β are functions of the static relative phases of the laser pulses.

6 Conclusion

In this paper, we have proposed a mechanism for the construction of a CNOT gate. This technique requires the use of one cavity and eleven pulses. It is robust against variations of amplitude and duration of the pulses and of the delay between the pulses. Moreover, it constitutes a decohence-free method in the sense that the excited atomic states with short life times are not populated in the adiabatic limit, and the cavity mode is negligibly populated during the process. This technique can be also an alternative to the composition of many elementary gates by a direct construction of specific gates, which could have potential applications for the fast realisation of some algorithms.

N.S. acknowledges support from the EU network QUACS under Contract No. HPRN-CT-2002-0039 and from La Fondation Carnot.

References

1. A. Barenco et al., Phys. Rev. Lett. **74**, 4083 (1994); D. DiVincenzo, Phys. Rev. A **51**, 1015 (1995); A. Barenco et al., Phys. Rev. A **52**, 3457 (1995)
2. A. Galindo, M.A. Martin-Delgado, Rev. Mod. Phys. **74**, 347 (2002)
3. C.H. Bennett et al., Phys. Rev. Lett. **70**, 1875 (1993)
4. N. Gisin, G. Ribordy, W. Tittel, H. Zbinden, Rev. Mod. Phys. **74**, 145 (2002)
5. Z. Kis, F. Renzoni, Phys. Rev. A **65**, 032318 (2002)
6. R. Unanyan, M. Fleischhauer, B.W. Shore, K. Bergmann, Opt. Commun. **155**, 144 (1998)
7. N.V. Vitanov, K.A. Suominen, B.W. Shore, J. Phys. B **32**, 4535 (1999)
8. T. Pellizari, S.A. Gardiner, J.I. Cirac, P. Zoller, Phys. Rev. Lett. **75**, 3788 (1995)
9. H. Goto, K. Ichimura, Phys. Rev. A **70**, 012305 (2004)
10. U. Gaubatz, P. Rudecki, S. Schiemann, K. Bergmann, J. Chem. Phys. **92**, 5363 (1990)
11. K. Bergmann, H. Theuer, B.W. Shore, Rev. Mod. Phys. **70**, 1003 (1998)
12. The set $\{U, CNOT\}$ is a universal set and the $CNOT$ gate can be obtained by the composition of two Hadamard gates and one C-$phase(\pi)$ gate
13. N. Sangouard, X. Lacour, S. Guérin, H.R. Jauslin, e-print `arXiv:quant-ph/0505163, http://xxx.lanl.gov`, Phys. Rev. A (to be published)
14. F. Wilczek, A. Zee, Phys. Rev. A **52**, 2111 (1984)
15. V.S. Malinovsky, D.J. Tannor, Phys. Rev. A **56**, 4929 (1997); N.V. Vitanov, Phys. Rev. A **58**, 2295 (1998); A.M. Steane, D.M. Lucas, Fortschr. Phys. **48**, 839 (2000)
16. N.V. Vitanov, M. Fleischhauer, B.W. Shore, K. Bergmann, Adv. At. Mol. Opt. Phys. **46**, 55 (2001)
17. J. McKeever, A. Boca, A.D. Boozer, J.R. Buck, H.J. Kimble, Nature **425**, 268 (2003)
18. R. Miller, T.E. Northup, K.M. Birnbaum, A. Boca, A.D. Boozer, H.J. Kimble, J. Phys. B **38**, S551 (2005)